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RESPONSE OF FREQUENCY DOMAIN IN GENERALIZED THERMOELASTICITY WITH TWO TEMPERATURES

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The paper is concerned with the time-harmonic deformation in a homogeneous, isotropic, generalized thermoelastic medium with two temperatures. The Hankel transform is employed to solve the boundary-value problem in the frequency domain in the context of two generalized theories of thermoelasticity (Lord and Shulman, Green and Lindsay). The inverse transform integral is evaluated by using the Romberg integration in order to obtain the results in the physical domain. The components of the stresses as well as the temperature and conductive temperature obtained in this manner are computed numerically. The effects of two temperatures are presented graphically.

Keywords: *generalized thermoelasticity, time harmonic, normal point force, conductive temperature, Hankel transform, Romberg integration.*

Introduction. Thermoelasticity with two temperatures is one of the nonclassical theories of thermomechanics of elastic solids. The main difference of this theory with respect to the classical one is a thermal dependence. During the last few decades, an intense amount of attention has been placed upon the theories of generalized thermoelasticity as they attempt to overcome the shortcomings of the classical coupled theory of thermoelasticity, i.e., infinite speed of propagation of thermoelasticity disturbances, unsatisfactory response of a solid body to short laser action, and poor description of thermoelastic behavior at low temperature. The two well-established theories of generalized thermoelasticity are given by Lord and Shulman (the LS theory) [1] and Green and Lindsay (the GL theory) [2].

Chen and Gurtin [3] and Chen, Gurtin, and Willams [4] have formulated a theory of heat conduction in deformable bodies which depends on two distinct temperatures: the conductive temperature ϕ and the thermodynamic temperature θ. Briefly, the conductive temperature ϕ and its two successive gradients at a given material point and time determine the internal energy, entropy, stress, heat flux, and thermodynamic temperature at that point and time. The presence of two distinct temperatures leads to a dependence on higher gradients, which results in a theory including mechanical effects. Boley and Tolin [5], while studying the transient coupled thermoplastic boundary-value problem in half-space, concluded that two temperatures and strains are the foundation for representation of the results in the form of the wave-pulse response which occurs instantaneously throughout the body. Chen, Gurtin, and Willams [6] have suggested that the difference in these two temperatures is proportional to the heat supply and the temperatures may be equal under certain condition for time-independent situations. However, for time-dependent problems, particularly for problems related to wave propagation, these two temperatures are, in general, different, regardless of the heat-supply presence.

Quintanilla [7, 8] proved some theorems in thermoelasticity with two temperatures. Youssef [9] formulated a generalized two-temperature theory taking into account the theory of heat conduction in deformable bodies. Later on, Youseff and Al-Lehaibi [10] and Youseff and Al-Harby [11] investigated various problems on the basis of the two-temperature thermoelasticity with relaxation time and showed that the obtained results are qualitatively different as compared to those in the case of one-temperature thermoelasticity.

The deformation at any point of a medium enables one to analyze the deformation fields around mining tremors and drilling crust of the Earth. This can contribute to the theoretical consideration of the seismic and volcanic

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sources due to taking account of the deformation fields in the entire volume surrounding the source region. The purpose of the present paper is to obtain expressions for the stresses and both temperatures on the basis of two theories of thermoelasticity with two temperatures, when the time-harmonic mechanical or thermal source is applied, by the integral transform technique. The present model is useful for understanding the nature of interaction between mechanical and thermal fields since most of the structural elements of heavy industries are often subjected to mechanical and thermal stresses at an elevated temperature. This study is also of geophysical interest, particularly in the investigation concerned with earthquakes and other phenomena in seismology and engineering.

Basic Equations. Following Youseff [9], the field equations and constitutive relations in a generalized thermoelastic body with two temperatures can be written as

$$(\lambda + 2\mu)\,\nabla\,(\nabla\cdot\mathbf{u}) - \mu\,(\nabla\times\nabla\times\mathbf{u}) - \beta\left(1 + \tau_1\frac{\partial}{\partial t}\right)\nabla\theta = \rho\,\frac{\partial^2\mathbf{u}}{\partial t^2}\,, \tag{1}$$

$$K^*\nabla^2\phi = \rho C^*\left(1 + \tau_0\frac{\partial}{\partial t}\right)\frac{\partial\theta}{\partial t} + \beta\theta_0\left(\frac{\partial}{\partial t} + \eta\tau_0\frac{\partial^2}{\partial t^2}\right)\nabla\mathbf{u}\,, \tag{2}$$

$$t_{ij} = \lambda u_{k,k}\delta_{ij} + \mu\,(u_{i,j} + u_{j,i}) - \beta\left(1 + \tau_1\frac{\partial}{\partial t}\right)\theta\delta_{ij}\,, \tag{3}$$

where

$$\theta = (1 - a\nabla^2)\,\phi\,,$$

$$\beta = (3\lambda + 2\mu)\,\alpha_t\,. \tag{4}$$

Here, $\eta = 1$, $\tau_1 = 0$, $a \neq 0$ for the LS theory with the two-temperature parameter and $\eta = 0$, $\tau_1 > 0$, $a \neq 0$ for the GL theory with such a parameter.

Formulation and Solution of the Problem. We consider an isotropic, homogeneous, thermoelastic solid in an initially undeformed state at temperature θ_0. The cylindrical polar coordinate system (r, φ, z) has its origin on the surface the $z = 0$ with z axis directed normally to the medium. As the problem considered is plane axisymmetric, the field component u_φ is equal to zero:

$$\mathbf{u} = (u_r, 0, u_z)\,, \tag{5}$$

and u_r, u_z, ϕ, and θ are independent of φ. A normal force or a thermal source is assumed to act at the origin of the cylindrical system of coordinates. The following dimensionless quantities are introduced:

$$r' = \frac{\omega_1}{c_1}r\,,\ \ z' = \frac{\omega_1}{c_1}z\,,\ \ u_r' = \frac{\omega_1}{c_1}u_r\,,\ \ u_z' = \frac{\omega_1}{c_1}u_z\,,\ \ t_{zr}' = \frac{t_{zr}}{\beta\theta_0}\,,\ \ t_{zz}' = \frac{t_{zz}}{\beta\theta_0}\,,\ \ \phi' = \frac{\phi}{\theta_0}\,,\ \ \theta' = \frac{\theta}{\theta_0}\,,$$

$$t' = \omega_1 t\,,\ \ \tau_0' = \omega_1\tau_0\,,\ \ \tau_1' = \omega_1\tau_1\,,\ \ a' = \frac{\omega_1^2}{c_1^2}a\,, \tag{6}$$

where

$$c_1^2 = \frac{\lambda + 2\mu}{\rho}\,,\ \ \omega_1 = \frac{\rho C^* c_1^2}{K^*}\,.$$

The dimensionless expressions connecting the displacement components $u_r(r, z, t)$ and $u_z(r, z, t)$ with the scalar potential functions $\psi_1(r, z, t)$ and $\psi_2(r, z, t)$ are given by

$$u_r = \frac{\partial \psi_1}{\partial r} + \frac{\partial^2 \psi_2}{\partial r \partial z}, \quad u_z = \frac{\partial \psi_1}{\partial z} - \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} \right) \psi_2 . \tag{7}$$

Applying the Hankel transform defined as

$$\tilde{f}(\xi, z, s) = \int_0^\infty f(r, z, s)\, r J_n(r\xi)\, dr \tag{8}$$

and assuming the time harmonic behavior $\exp(i\omega t)$ in Eqs. (1)–(2), after using Eqs. (5)–(7) (omitting primes for convenience) and eliminating ψ_1, ψ_2, ϕ, and θ from the resulting expressions, we obtain

$$\left(\frac{d^4}{dz^4} + N \frac{d^2}{dz^2} + P \right) \tilde{\psi}_1 = 0 , \tag{9}$$

$$\left(\frac{d^2}{dz^2} + \lambda_3^2 \right) \tilde{\psi}_2 = 0 . \tag{10}$$

Here

$$N = \frac{a_{11}(a_1 + a_2) + a_9 a_8 - a_6 a_{10} + a_7 a_{12}}{a_9 (a_1 + a_2) - a_7 a_{10}}, \quad P = \frac{a_{11} a_8 + a_6 a_{12}}{a_9 (a_1 + a_2) - a_7 a_{10}},$$

$$a_1 = \frac{\lambda + \mu}{\rho c_1^2}, \quad a_2 = \frac{\mu}{\rho c_1^2}, \quad a_3 = -\frac{\beta \theta_0}{\rho c_1^2}, \quad a_4 = \frac{\rho C^* c_1^2}{K^* \omega_1}, \quad a_5 = \frac{\beta c_1^2}{K^* \omega_1},$$

$$a_6 = a_3 (1 - a\xi^2) f_1 , \quad a_7 = - a a_3 f_1 , \quad a_8 = (a_1 + a_2)\, \xi^2 + \omega^2 , \quad a_9 = 1 + a a_4 f_2 ,$$

$$a_{10} = - a_5 f_3 , \quad a_{11} = \xi^2 - a_4 f_2 (1 - a\xi^2) , \quad a_{12} = a_5 (i\omega - \eta\omega^2)\, \xi^2 , \quad \lambda_3^2 = \xi^2 + \frac{\omega^2}{a_2} ,$$

where

$$f_1 = 1 + i\omega\tau_1 , \quad f_2 = i\omega - \omega^2 \tau_0 , \quad f_3 = i\omega - \eta\omega^2 .$$

The roots of Eq. (9) are $\pm\lambda_k$ ($k = 1, 2$) and the roots of Eq. (10) are equal to $\pm\lambda_3$. With the use of the radiation conditions according to which $\tilde{\psi}_1, \tilde{\psi}_2, \tilde{\phi} \to 0$ as $z \to \infty$, the solutions of Eqs. (9)–(10) can be written as

$$\tilde{\psi}_1 = A_1 \exp(-\lambda_1 z) + A_2 \exp(-\lambda_2 z) , \tag{11}$$

$$\tilde{\psi}_2 = A_3 \exp(-\lambda_3 z) , \tag{12}$$

$$\tilde{\phi} = d_1 A_1 \exp(-\lambda_1 z) + d_2 A_2 \exp(-\lambda_2 z) , \tag{13}$$

where

$$d_1 = \frac{(a_1 + a_2)\,\lambda_1^2 + a_8}{a_6 + a_7\lambda_1^2}\,, \quad d_2 = \frac{(a_1 + a_2)\,\lambda_2^2 + a_8}{a_6 + a_7\lambda_2^2}\,.$$

Boundary Conditions. For mechanical and thermal sources acting on the surface, the boundary conditions are the following:

$$t_{zz} = -P_1\,(r, t)\,, \tag{14}$$

$$t_{zr} = 0\,, \tag{15}$$

$$\theta = P_2\,(r, t)\,. \tag{16}$$

Particular Cases. For the *concentrated time-harmonic normal source and the thermal source* of unit magnitude, we have

$$(P_1, P_2)\,(r, t) = \frac{\delta\,(r)\,\exp\,(i\omega t)}{2\pi r} \tag{17}$$

with

$$(\hat{P}_1, \hat{P}_2)\,(\xi, t) = \frac{1}{2\pi}\,. \tag{18}$$

For the *time-harmonic normal source of unit magnitude over a circular region*, we can write

$$(P_1, P_2)\,(r, t) = \frac{\mathrm{H}\,(a_1 - r)\,\exp\,(i\omega t)}{\pi a_1^2} \tag{19}$$

with

$$(\hat{P}_1, \hat{P}_2)\,(\xi, t) = \frac{J_1\,(a_1\xi)}{2\pi}\,. \tag{20}$$

Using Eqs. (3)–(7) in the boundary conditions (14)–(16), applying the Hankel transform defined by (8), and substituting the values of $\tilde{\psi}_1$, $\tilde{\psi}_2$, and $\tilde{\phi}$ from Eqs. (11)–(13) into the resulting equations, we obtain the following expressions for the components of the displacement and stress as well as for the temperature and conductive temperature:

$$\tilde{u}_z = -\frac{1}{\Delta}\Big[-\lambda_1\Delta_1\,\exp\,(-\lambda_1 z) + \lambda_2\Delta_2\,\exp\,(-\lambda_2 z) + \xi^2\Delta_3\,\exp\,(-\lambda_3 z)\Big]\exp\,(i\omega t)\,, \tag{21}$$

$$\tilde{u}_r = \frac{1}{\Delta}\Big[\xi\,\Big\{\Delta_1\,\exp\,(-\lambda_1 z) - \Delta_2\,\exp\,(-\lambda_2 z) + \lambda_3\Delta_3\,\exp\,(-\lambda_3 z)\Big\}\Big]\exp\,(i\omega t)\,, \tag{22}$$

$$\tilde{t}_{zz} = \frac{1}{\Delta}\Big[s_1\Delta_1\,\exp\,(-\lambda_1 z) + s_2\Delta_2\,\exp\,(-\lambda_2 z) + s_3\Delta_3\,\exp\,(-\lambda_3 z)\Big]\exp\,(i\omega t)\,, \tag{23}$$

$$\tilde{t}_{zr} = \frac{1}{\Delta}\Big[-s_4\Delta_1\,\exp\,(-\lambda_1 z) + s_5\Delta_2\,\exp\,(-\lambda_2 z) + s_6\Delta_3\,\exp\,(-\lambda_3 z)\Big]\exp\,(i\omega t)\,, \tag{24}$$

$$\tilde{\theta} = \frac{1}{\Delta}\Big[s_7\Delta_1\,\exp\,(-\lambda_1 z) + s_8\Delta_2\,\exp\,(-\lambda_2 z)\Big]\exp\,(i\omega t)\,, \tag{25}$$

$$\tilde{\phi} = \frac{1}{\Delta}\Big[-d_1\Delta_1 \exp(-\lambda_1 z) + d_2\Delta_2 \exp(-\lambda_2 z)\Big] \exp(i\omega t)\,, \tag{26}$$

where

$$\Delta = \begin{vmatrix} s_1 & s_2 & s_3 \\ s_4 & s_5 & s_6 \\ s_7 & s_8 & 0 \end{vmatrix},$$

Δ_i for $i = 1, 2, 3$ is obtained from Δ by replacing the corresponding column with $|-\hat{P}_1 \ 0 \ \hat{P}_2|^T$, and

$$s_1 = e_1\lambda_1^2 + e_2\xi^2 - f_1\Big[(1 - a\xi^2)\,d_1 - ad_1\lambda_1^2\Big],$$

$$s_4 = 2\xi\lambda_1\,, \quad s_5 = 2\xi\lambda_2\,, \quad s_6 = \xi\,(\xi^2 - \lambda_3^2)\,, \quad s_7 = d_1\Big[(1 - a\xi^2) - a\lambda_1^2\Big],$$

$$s_8 = d_2\Big[(1 - a\xi^2) - a\lambda_2^2\Big], \quad e_1 = \frac{\lambda + 2\mu}{\beta\theta_0}\,, \quad e_2 = \frac{\lambda}{\beta\theta_0}\,.$$

To obtain the expressions for the concentrated normal force and the thermal source applied over a circular region, it is necessary to use the relations for $\hat{P}_1$, $\hat{P}_2$ from Eqs. (19) and (20) in Eqs. (21)–(26).

Special Cases. The following cases could be recognized:

- $\tau_1 = 0$, $\eta = 1$, and $a \neq 0$ — two-temperature theory of thermoelasticity with one relaxation parameter (TLS theory);
- $\tau_1 > 0$, $\eta = 0$, and $a \neq 0$ — two-temperature theory of thermoelasticity with two relaxation parameters (TGL theory);
- $\tau_1 = \tau_0 = 0$ and $a \neq 0$ — coupled thermoelasticity with two temperatures;
- $\tau_1 = 0$, $\eta = 1$, $a = 0$ and $\tau_1 > 0$, $\eta = 0$, $a = 0$ — classical one-temperature LS and GL theories, respectively.

Inversion of the Transforms. To obtain the solution of the problem in the physical domain, we should invert the transforms in Eqs. (21)–(26) for the LS and GL theories. These expressions are functions of z and parameters of the Hankel transforms s and ξ, hence they are of the form $\tilde{f}(\xi, z, s)$. To get the function $f(r, z, t)$ in the physical domain, first we invert the Hankel transform using the expression

$$f(r, z, s) = \int_0^\infty \xi \tilde{f}(\xi, z, s)\, J_n(r\xi)\, d\xi\,. \tag{27}$$

The last step is to calculate the integral in Eq. (27). The method for its evaluation which is described by Press et al. [12] involves the use of the Romberg integration with adaptive step size. This also enables one to refine the results successively using the extended trapezoidal rule followed by extrapolation of the results to the limit, when the step size tends to zero.

Numerical Results and Discussion. The material chosen for the calculation is copper (see [13]), whose characteristics are as follows:

$$\lambda = 7.76\cdot 10^{10}\ \text{N}\cdot\text{m}^{-2}\,, \quad \mu = 3.86\cdot 10^{10}\ \text{N}\cdot\text{m}^{-2}\,, \quad \rho = 8954\cdot 10^{3}\ \text{kg}\cdot\text{m}^{-3}\,,$$

$$\alpha_t = 1.78\cdot 10^{-5}\ \text{K}^{-1}\,, \quad T = 293\ \text{K}\,, \quad K^* = 386\ \text{W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}\,, \quad C^* = 383.1\ \text{J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}\,,$$

and the dimensionless relaxation times are taken as $\tau_0 = 0.02$, $\tau_1 = 0.04$.

The variations of the normal stress t_{zz}, tangential stress t_{zr}, temperature θ, and conductive temperature ϕ with distance r are presented in Figs. 1–4 for the TLS and TGL theories (i.e., respectively for the theories of Lord–Shulman

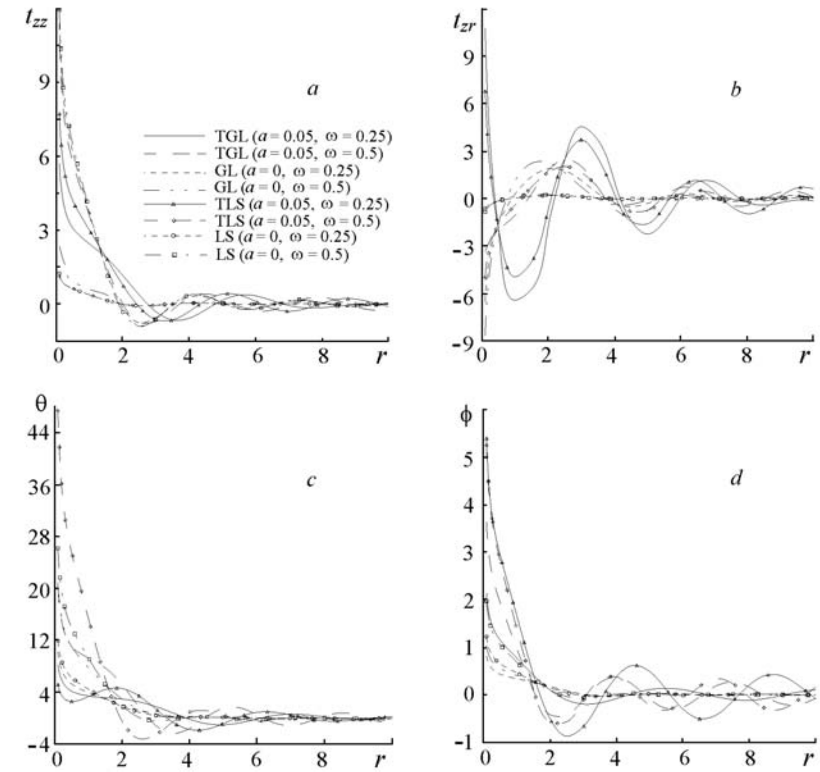

Fig. 1. Variation of the dimensionless normal stress (a), tangential stress (b), temperature (c), and conductive temperature (d) with distance for the concentrated normal force.

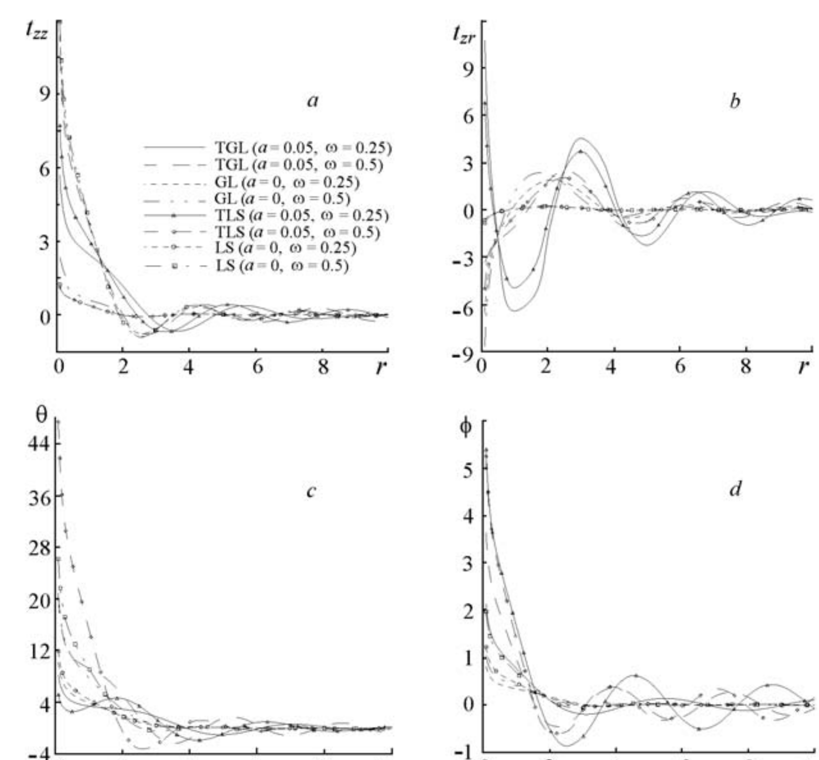

Fig. 2. Same as in Fig. 1 for the normal force over a circular region.

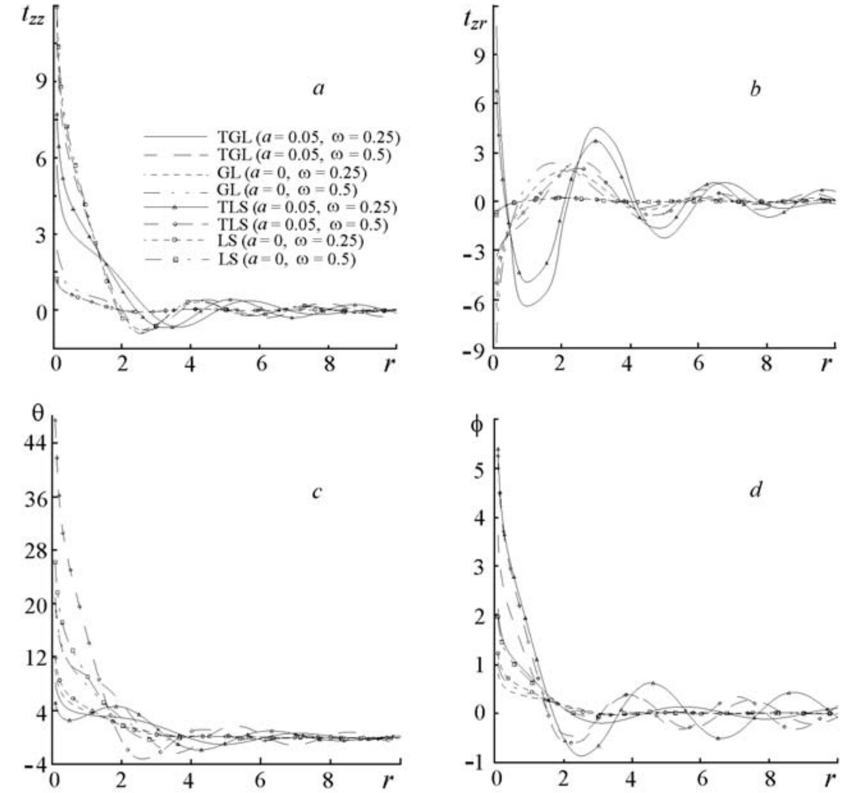

Fig. 3. Same as in Fig. 1 for the thermal point source.

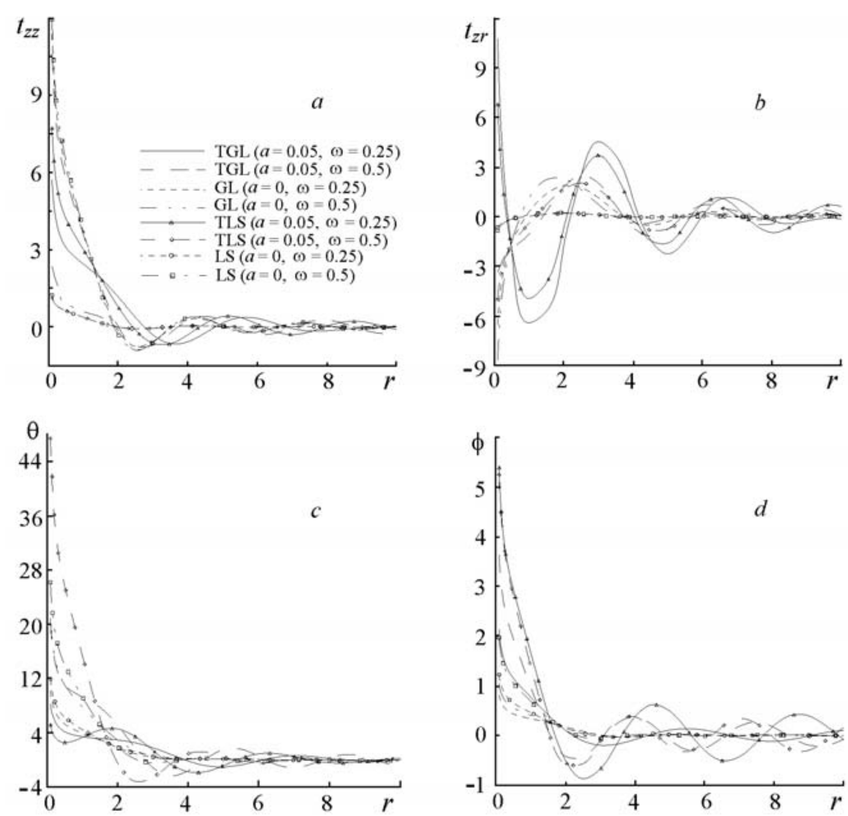

Fig. 4. Same as in Fig. 1 for the thermal source over a circular region.

and Green–Lindsay with two temperatures) at $\omega = 0.25$ and 0.5 and $a = 0.05$ as well as for the LS and GL theories, i.e., for the classical one-temperature theories at $a = 0$. We shall dwell on some peculiarities of these dependences.

Mechanical Forces Acting on the Surface. For the *concentrated normal force* it is evident from Fig. 1a that the values of the normal stress t_{zz} at $\omega = 0.25$ and $a = 0.05$ in the range $0 \le r < 2$ for the TLS theory are greater than those for the TGL theory. With a further increase in r, oscillatory patterns for both theories of thermoelasticity (TLS and TGL) take place. For $\omega = 0.25$ and $a = 0.05$ the trends are opposite in comparison with the first case mentioned.

It is seen from Fig. 1b that the tangential stress t_{zr} near the point of the source application at $\omega = 0.25$ and $a = 0.05$ for the TGL theory is greater than that for the TLS theory. For $\omega = 0.5$ in the region $0 \le r < 1$ the values of t_{zr} for the TLS and LS theories are greater than those for the TGL and GL theories, respectively.

Figure 1c shows the temperature distribution θ. For $\omega = 0.25$ and $a = 0.05$ the values of θ for the TLS theory are smaller than for the TGL theory as $0 \le r < 1$, while the opposite behavior is noted at $a = 0$. For $\omega = 0.5$ and $a = 0$ the results of both theories essentially coincide everywhere over the interval.

It is seen from Fig. 1d that the trend in the variation of the conductive temperature ϕ is opposite to that observed for θ.

Figure 2a–d shows the characteristics mentioned for the *normal force over a circular region* for both theories of thermoelasticity.

Thermal Source Acting on the Surface. For the *thermal point source*, Fig. 3a shows the variations of t_{zz}. The values for both of the theories at $\omega = 0.25$ and $\grave{a} = 0.05$ increase sharply in the range $0 \le r < 1.5$, and the values for the TLS theory are greater. For both values of ω the trends at $a = 0$ and $a = 0.05$ are opposite.

The variations of the tangential stress t_{zr} are shown in Fig. 3b. For both values of ω and $a = 0.05$, the values of t_{zr} in the range $0 \le r < 1$ for the TGL theory are greater.

The temperature distribution θ is shown in Fig. 3c. For all the cases considered, the values of θ near the point of the source application for the TGL theory are greater than those for the TLS theory.

Figure 3d shows the variations of the conductive temperature ϕ. The trends in these variations for $\omega = 0.5$ and both values of a as well as for $\omega = 0.25$ and $a = 0.05$ are the same as those for θ, but for $\omega = 0.25$ and $a = 0$ the trends are opposite.

For the *thermal point source over a circular region*, the variation of the normal stress t_{zz} is seen in Fig. 4a. For $\omega = 0.25$ and $a = 0.05$ in the range $0 \le r < 1.5$ the values of t_{zz} for the TLS theory are greater than those for the TGL theory, while the classical one-temperature theories of thermoelasticity show the opposite behavior. At $\omega = 0.5$ and $a = 0.05$, the trend is opposite to the first case mentioned.

The variation of the tangential stress t_{zr} is shown in Fig 4b. Near the point of the source application, for $\omega = 0.25$ and $a = 0.05$ there is a small difference between the results for both theories, but with an increase in r they demonstrate opposite oscillatory patterns. For $\omega = 0.5$ at both values of a near the point of the source application the values of t_{zr} are greater for both theories of thermoelasticity, but further the difference becomes negligible.

Figure 4c depicts the temperature distributions. For $\omega = 0.25$ at $a = 0.05$ and $a = 0$ the values of θ for the TLS theory are greater. The same is observed for the classical one-temperature theory with significant difference in the magnitudes, whereas for $\omega = 0.5$ both values of a lead to a small difference between the results.

Figure 4d shows the variations of the conductive temperature ϕ. For both values of ω in the range $0 \le r < 1.5$ the values of ϕ for the TGL theory are greater than those for the TLS theory, whereas for $a = 0$ a reverse pattern is observed.

It is seen that all the figures show that the two-temperature parameter has a significant effect on the results at small r. Further, beginning from some value of r, the dependences are characterized by oscillations with decreasing magnitudes.

Conclusions. The generalized theories of thermoelasticity of Lord–Shulman and Green–Lindsay were used to solve the problem. The effect of two temperatures is significant for the normal stress, tangential stress, temperature, and conductive temperature. It is observed from the figures that the trends in the variations of the characteristics mentioned are similar with difference in their magnitude when the concentrated force and the force over a circular region are applied, whereas the trends are different when the point thermal source and the source over a circular region are applied. As disturbances travel through the medium, they are characterized by sudden changes, which results in an inconsistent/nonuniform pattern.

NOTATION

a, two-temperature parameter; a_1, radius of a circular region; C^*, specific heat; J, Bessel function; K^*, thermal conductivity; P_1, normal force, P_2, thermal source; r, φ, z, coordinates; s, ξ, parameters of the Hankel transform; t, time; t_{ij}, components of the stress tensor; **u**, displacement vector; α_t, linear thermal expansion coefficient; η, constant; θ, temperature; θ_0, reference temperature; λ, μ, Lamé constants; ρ, density; τ_0, τ_1, relaxation times; ϕ, conductive temperature; ψ_1, ψ_2, scalar potential functions; ω, circular frequency.

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